Estimating Slope and Level Change in N=1 Designs

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ABSTRACT
The study proposes a new procedure referred to as Slope and Level Change (SLC) for separately estimating slope change and level change between two adjacent phases in single-case designs, for instance between a baseline phase (denoted by A) and a treatment phase (denoted by B). Firstly, SLC eliminates baseline trend from the data series using a differencing method, instead of the commonly used regression. Secondly, the method used for estimating trend is also used for estimating slope change. After the slope change has been controlled for, the pure level change is estimated. The steps necessary to obtain the estimates are presented in detail, explained, and illustrated by means of a fictitious data example. A simulation study is carried out to explore the bias and precision of the estimators and compare them to an analytical procedure matching the data simulation model. The results suggest that the SLC estimates are unbiased for all levels of autocorrelation tested and control effectively for trend. An R code was developed to make the application of the procedure automatic.

Author Keywords
Single-case designs, level change, slope change, trend, autocorrelation.

INTRODUCTION

The need to complement the widely extended *p* values with measures of the strength of association between an independent and a dependent variable has been highlighted decades ago [6,13], since the former only focus on the null hypothesis. In recent years it has become widely accepted that not only group-design but also single-case (N=1) design studies need to provide scientific evidence on the interventions applied [9,15]. The use of magnitude of effect measures would allow subsequent N=1 studies to have a solid foundation and would also make possible to perform meta-analyses.

In N=1 studies, effect sizes have been commonly expressed as amount of variability in the behavior accounted for by treatment introduction [16] or as the amount of overlap between data pertaining to different conditions [14]. Evidence suggests that the procedures based on regression analysis perform less than optimally [3,4]. Specifically, they do not distinguish sufficiently between presence and absence of treatment effect and the R^2 values they provide are distorted when data are sequentially related. The procedures based on data overlap do not seem to be that affected by autocorrelation, but the presence of general trend in data remains problematic [10].

The present study arises from the need to develop a procedure that quantifies precisely the amount of behavioral change present in data. In addition, SLC responds to the call for two separate estimates of level and slope change in single-case studies [2].

PROCEDURE PROPOSED

Rationale

The objective of SLC is to estimate level change and slope change eliminating baseline data linear trend whenever it is present. It is conjectured that the procedure may also deal with positive serial dependence, since the presence of large positive autocorrelation in data can be represented by an upward or a downward trend. In contrast, when measurements are negatively autocorrelated, data present greater variability rather than trend. The slope and level change estimates obtained for the detrended data express the shifts in terms of the measurement units. For instance, if the frequency of behavior is measured, a slope change of 3 would mean that at each measurement time during phase B the experimental unit produces an average of three behaviors more than in the previous observation point. A level change of 3 would imply that with the introduction of the intervention (i.e., with the change in phase) the experimental unit produces an average of 3 behaviors more in the treatment phase than in the baseline phase. It has to be remarked that this average change in level is computed after eliminating the previously estimated slope change.

Steps Required to Compute the Procedure

Since SLC is designed to control general trend prior to assessing intervention effectiveness, an initial data correction step involves eliminating phase A (baseline) trend from data. Trend is estimated only for the baseline phase, since in that way it is possible to avoid confusion between trend and potential intervention effects taking place in phase B (treatment) [1]. The phase A trend is estimated as the mean of the differenced phase A measurements. These steps can be expressed as differencing, Equation (1), and averaging the differenced data, Equation (2):

$$\Delta A_t = A_{t+1} - A_t \qquad \text{Equation (1)}$$

$$\overline{\Delta A} = \sum_{t=1}^{n_A-1} \Delta A_t \Big/ (n_A - 1) = \widehat{\beta}_A \qquad \text{Equation (2)}$$

where A_t represents the original phase A measurement at time *t*, ΔA_t represents the differenced phase A data, and n_A is the number of observations in the baseline phase. Consider a fictitious example of data series consisting of the following values 1, 2, 3, 4, and 5 for phase A, and 7, 9, 11, 13, and 15 for phase B. The phase A trend estimate $\widehat{\beta}_A$ is used to correct data (i.e., remove trend) and the abovementioned measurements the differenced phase A data will contain only 1's and their mean (the estimate of trend) would also be one. Trend is eliminated from A data via Equation (3), and from phase B data via Equation (4):

$$\widetilde{A}_t = A_t - \widehat{\beta}_A \cdot t \qquad \text{Equation (3)}$$

$$\widetilde{B}_t = B_t - \widehat{\beta}_A \cdot t \qquad \text{Equation (4)}$$

where $\widetilde{A}_i$ and $\widetilde{B}_i$ represents detrended phase A and phase B data, respectively, and *t* is the measurement time. After the correction the phase A data will be all 0's and the phase B data: 1, 2, 3, 4, 5. This method for detrending data has been shown to be useful for dealing not only with trend but also with autocorrelation [12].

The second step involves estimating slope change as the trend present in the phase B data, from which baseline trend has already been removed. The detrended phase B data is differenced according to Equation (5):

$$\Delta\widetilde{B}_t = \widetilde{B}_{t+1} - \widetilde{B}_t \qquad \text{Equation (5)}$$

The differenced and already detrended phase B data of the example consists of four 1's. The mean of these differenced measurements is computed as shown in Equation (6) obtaining the phase B trend estimate $\widehat{SC}$

$$\overline{\Delta\widetilde{B}} = \sum_{t=n_A+1}^{n_A+n_B-1} \Delta\widetilde{B}_t \Big/ (n_B - 1) = \widehat{SC} \qquad \text{Equation (6)}$$

where n_B is the number of observations in the treatment phase. The average value is assumed to represent an estimation of slope change, $\widehat{SC}$, considering that the phase A trend has been previously removed. In the fictitious case the average of four 1's is 1 and so the slope change is estimated as 1.

The third step consists in the estimation of level change. Firstly, the already estimated change in slope is eliminated from the treatment phase data, without removing the intercept. That is, the phase B slope is eliminated from the detrended phase B data, while maintaining potential shifts taking place at time n_A+1. This is achieved through Equation (7):

$$\widetilde{\widetilde{B}}_t = \left(\widetilde{B}_t - \widehat{SC} \cdot (t-1)\right) \qquad \text{Equation (7)}$$

When from the detrended phase B data (1, 2, 3, 4, and 5) the slope change of 1 is removed, the following values are obtained 1, 1, 1, 1, and 1 and represented the phase B data with trend and slope change eliminated.

Level change (LC) is estimated subtracting the detrended baseline data mean from the detrended and slope-change-controlled treatment data mean, as shown in Equation (8):

$$\widehat{LC} = \overline{\widetilde{\widetilde{B}}} - \overline{\widetilde{A}} = \sum_{t=n_A+1}^{n_A+n_B} \widetilde{\widetilde{B}}_t \Big/ n_B - \sum_{t=1}^{n_A} \widetilde{A}_t \Big/ n_A \qquad \text{Equation (8)}$$

Through this expression level change is estimated in terms of average level of behavior in both phases. The phase A data after removing trend is represented by five 0's, while the phase B data after removing trend and slope change consists of five 1's, so level change is equal to 1−0= 1. Both the slope and the level change estimates represent precisely the parameters used to construct (without random variability) the fictitious data set,.

The procedure described is not restricted to AB designs and can be applied to any combination of a baseline and treatment phase which is included in more complex design structures (e.g., multiple-baselines designs, ABAB designs).

METHOD

Data Generation

In order to test the performance of SLC for data series with random fluctuations and with known features, Monte Carlo simulation was used. The design structure studied was AB with the following series lengths (n_A + n_B): 5+5, 5+10, 7+8, 10+10, 15+15, 20+20. The model used for data generation was the one presented in [8]: $y_t = \beta_0 + \beta_1 \cdot T_t + \beta_2 \cdot LC_t +$

$\beta_3 \cdot SC_t + \varepsilon_t$, where y_t is the value of the dependent variable at moment t, β_0 is the intercept (set to zero), β_1, β_2, and β_3 are the coefficients associated with trend, slope change, and level change, respectively, T_t is a dummy time variable (taking values from 1 to $n_A + n_B$), LC_t is a dummy variable for level change (equal to 0 for phase A and to 1 for phase B), SC_t is a dummy variable for slope change being equal to 0 for phase A, and taking values from 0 to (n_B-1) for phase B, and ε_t is the error term. The beta parameters related to trend, level and slope change were set to 1 and 10 to represent a small and a large effect, respectively.

The error term (ε_t) was generated following the models assumed to represent adequately the greater part of behavioral data [7]: a) the first-order autoregressive model AR(1) $\varepsilon_t = \varphi_1 \cdot \varepsilon_{t-1} + u_t$, with φ_1 ranging from –0.9 to 0.9 with steps of .1; and b) the first-order moving average model MA(1) $\varepsilon_t = u_t - \theta_1 \cdot u_{t-1}$ with 19 values of θ_1, which according to the relationship between MA(1) and AR(1) processes leads to parameter φ_1 values ranging from 0.4972 to −0.4972.

For both models the random variable u_t was generated following three distributions (exponential, normal, and uniform) in order to study the effect of skewness and kurtosis on the performance of SLC.

Data Analysis

For each experimental condition the mean and variance of the estimators was computed on the basis of 100,000 samples. The bias of the estimators was obtained as the difference between the simulation parameters and the estimates for slope and level change. The variance of the estimators was computed as an indicator of efficiency and a comparison was performed between SLC and a simultaneous multiple regression (SMR) procedure whose model corresponds exactly to the data parameters used [8].

Simulation

The simulation was carried out by means of Fortran programs – one for each combination of data generation process [AR(1) or MA(1)] and distribution of the random variable u_t term (exponential, normal, or uniform). Each program consisted of the following steps: 1) series length selection; 2) specification of the value of φ_1 or θ_1; 3) specification of the β_1, β_2, and β_3 parameters; 4) 100,000 iterations of steps 5 to 8; 5) generate the error term ε_t according to the generation process and the u_t distribution; 6) obtain the values for the dummy variables T, LC, and SC; 7) obtain y_t; 8) apply SLC and SMR; 9) obtain the mean and variance of the estimates for both estimators of each procedure.

RESULTS

For both independent and serially related data with and without general trend, SLC and SMR provide unbiased estimates of the treatment effect. Therefore, the procedure proposed controls effectively for trend and is also unaffected by the lack of independence in data. This finding is general for both data generation processes and for the three error distributions.

Regarding the variability of the SLC estimators, it is increased by high negative autocorrelation, when data generated by a MA(1) process, and when there are less data points in the series.

CONCLUSION

SLC is unbiased both for first-order autoregressive and moving average processes and regardless of the distributional shape of the random variable. The SLC estimators are generally more efficient (i.e., less variable) than the SMR ones for positively autocorrelated data. Considering the large effects typical present in single-case data [5], the variability of the estimators does not seem excessive.

SLC is less efficient than the technique matching perfectly the data simulation model only for high negative serial dependence and when treatment effect is expressed as level change. Regarding the former, high negative autocorrelation is not frequent in N=1 data [12], while in relation to the latter it has to be remarked that in psychological studies an abrupt and sustained (level) change in the behavior is less likely to occur than a progressive change representing a gradual improvement of the individual or group treated. As the performance of the procedure proposed was tested using data simulated using Monte Carlo methods, further evidence on the ease and meaningfulness of SLC's application can be provided using real behavioral data.

Further research is needed in order to explore whether the data correction present in SLC can attenuate the effect of nonlinear trends. Future efforts may also focus on estimating the sampling distribution of the slope and level change estimators, due to its utility for obtaining statistical significance and, more importantly, confidence intervals.

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